



St Peter's follow the White Rose scheme of work and this document demonstrates the progression in formal written calculations. Written Calculation Progression links the key concrete experiences with pictorial and abstract representations (written symbolic and spoken). This supports children to move with confidence and deep conceptual understanding through each strand of calculation. Each operation is also broken down into skills with different models and images that could effectively be used to teach that concept to all children. White Rose provides end of block assessments to assess learning and end of term assessments are also used. Mental maths is taught daily using Schofield and Simms.

Concrete Manipulatives

Concrete manipulatives are objects that can be touched and moved by children to introduce, explore, or reinforce a mathematical concept. They provide a vehicle to help children make sense of complex, symbolic and abstract ideas through exploration and manipulation. Furthermore, they support the development of internal models and help build stronger memory pathways.

Pictorial (including jottings)

The act of translating the concrete experience into a pictorial representation helps focus attention on what has happened and why. This supports deeper understanding and a stronger imprint on memory. Pictorial representations are more malleable than concrete resources and, once understanding is secured, allow exploration of complex problems that may be challenging to reproduce with manipulatives.

Abstract - Written

The aim, within this policy, is for compacted forms of notation. These have developed through the history of mathematics. Explicit individual steps in procedure are hidden or they have been shortcut. The informal and expanded methods expose all the intermediate steps, replicating thought processes more closely and support understanding prior to compaction.

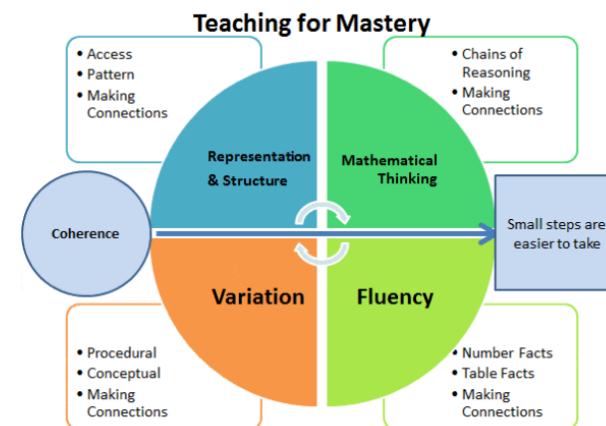
Abstract - Spoken

Learning to use the correct mathematical vocabulary is vital for the development of mathematical proficiency. The ability to articulate accurately allows children to communicate and build meaning. Ideas become more permanent. This can be scaffolded effectively using precise mathematical talk.

As part of the school's professional development, staff at St Peter's are currently working towards a mastery approach in maths (2021-22).

KS1 – Mastering Number

KS2 – Whole school -Teaching for mastery led by Mr Ormerod and Mrs Dowson



Addition

Year 1

Addition

National Curriculum

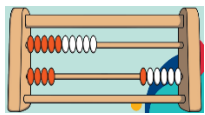
Given a number, identify one more

Add one-digit and two-digit numbers to 20, including zero.

Represent and use number bonds and related subtraction facts within 20

Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as $7 = _ - 9$.

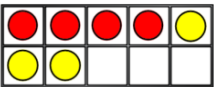
Concrete



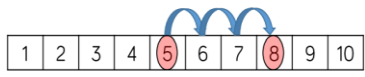
Rekenreks to build number sense and support counting, subitising and essential maths concepts such as doubling.



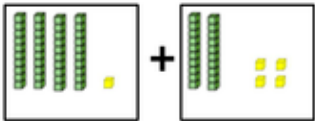
Numicon and number shapes for counting, ordering, patterns, number bonds and calculating. 4 and 3 more



Tens frames for number recognition and augmentation. Count on or count all.



Number track or hundred square to support counting on.

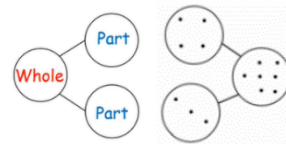


Dienes to support understanding of place value and number size.

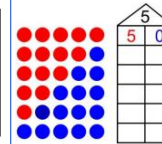


Using dice and spinners to promote enjoyment as well as number recognition.

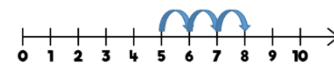
Pictorial (representations) Draw out the connections between models.



Part Whole
Count on or count all.

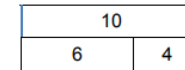
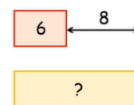
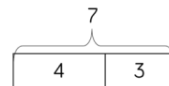


Bonds

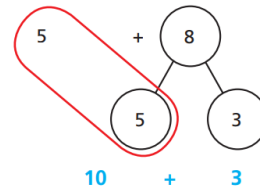


Number lines to reinforce counting on from the largest number.

Bar models support children in representing a problem or calculation.



$$\begin{aligned} 6 + 4 &= 10 \\ 4 + 6 &= 10 \\ 10 - 4 &= 6 \\ 10 - 6 &= 4 \end{aligned}$$



Add by making 10.
Needs strong number bond skills!

Abstract – contextualise the number sentence

$$\square + \square = \square$$

$$\square = \square + \square$$

Representing number sentences in different ways and continuing with the mathematical talk e.g. 4 is a part, 5 is a part, 9 is the whole.

$$\begin{aligned} 5 + 5 &= 10 \\ 5 + 5 &= 8 \\ 2 + 5 &= 5 + 3 \end{aligned}$$

Use $<$, $>$ and $=$ signs to show inequalities.

$$\begin{aligned} \square + \square &= 8 \\ \square + \square &= 8 \\ 8 - \square &= \square \\ 8 - \square &= \square \end{aligned}$$

Create related number sentences 'Fact Families'.

Mathematical Talk

Addition can be done in any order

This is a part, this is a part, this is the whole.

__ones and __ones equal __ones

10 ones is equal 1 ten

49 has __ tens and __ ones

41 has 4 tens and 1 one

Ready to progress expectation

Count within 100, forwards and backwards, starting with any number

Compose numbers to 10 from 2 parts, and partition numbers to 10 into parts, including recognising odd and even numbers.

Reason about the location of numbers to 20 within the linear number system, including comparing using $<$, $>$ and $=$

Read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs.

Develop fluency in addition and subtraction facts within 10

Year 2

Addition

National Curriculum

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two, two-digit numbers

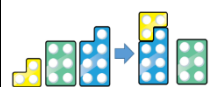
Solve problems with addition and subtraction

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100

Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Concrete

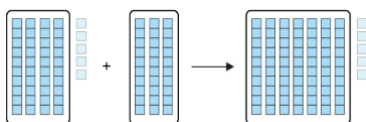


Adding 3 one-digit numbers. Children identify bonds to 10, doubles or near doubles.

Regrouping



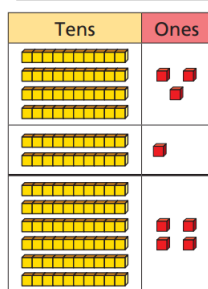
Unitising – one-digit calculations



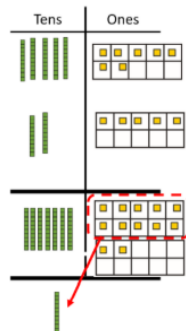
Hundred square



No regrouping

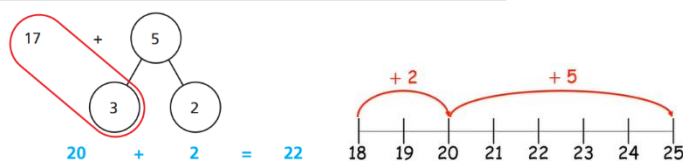


Regrouping

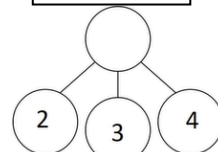


Pictorial (representations)

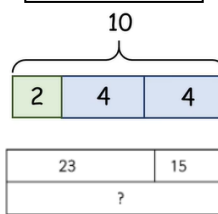
Adding a two-digit and one-digit number crossing ten.



Part-whole

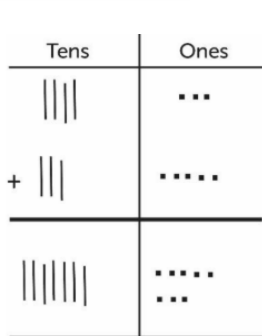


Representing

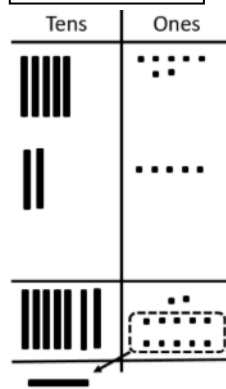


Scaffold - Draw the tens and ones

No regrouping



Regrouping



Abstract

Variation – What's the same? What's different?

23 + 10 23 + 20 23 + 30 23 + 40

Making bonds or doubles

$$4 + 7 + 6 = 10 + 7$$

No regrouping

$$\begin{array}{r} 40 \ 3 \\ + 30 \ 5 \\ \hline 70 \ 8 \end{array}$$

T	O
4	6
+ 1	3
5	9

Regrouping

$$\begin{array}{r} 50 + 7 \\ + 20 + 5 \\ \hline 80 + 12 \\ \hline 10 \end{array}$$

T	O
2	5
+ 7	0
9	5

T	O
4	6
+ 1	5
5	1

Mathematical Talk

Addition can be done in any order

When we add tens the ones digit stays the same

12 ones is regrouped to equal 1 ten and 2 ones

The sum of __ ones and __ ones is __ ones

The sum of __ tens and __ tens is __ tens

And... __ tens and __ ones is equal to __

The sum of __ ones and __ ones is __ ones, this is regrouped into __ ten and __ ones

Ready to progress expectation

Add and subtract across 10

Recognise the subtraction structure of 'difference' and answer questions of the form, "How many more...?"

Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract only ones or only tens to/from a twodigit number

Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract any 2 two-digit numbers.

Year 3 Addition

National Curriculum

Add and subtract numbers mentally, including:

- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Concrete

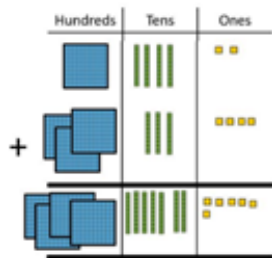
Unitising



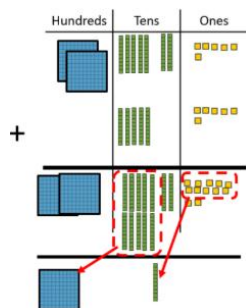
Regrouping – ten ones for one ten or ten tens for one hundred



No regrouping

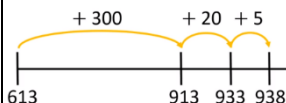
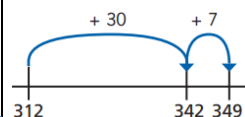


Regrouping

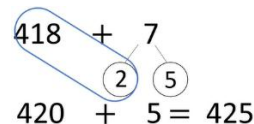


Pictorial

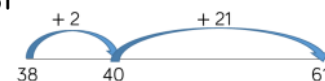
Not crossing



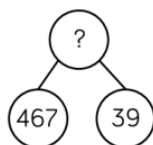
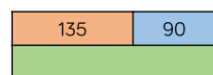
Crossing ten - bridging



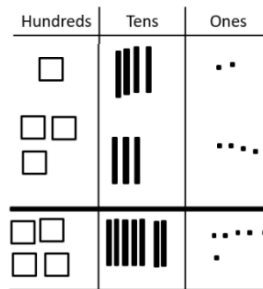
$$38 + 23 = 61$$



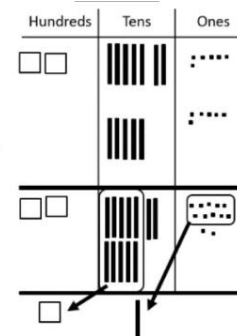
Representing



No regrouping



Regrouping



Abstract - Children must understand how it links to place value at each stage.

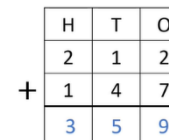
Variation –
What's the same?
What's different?

$$222 + 6 = \square$$

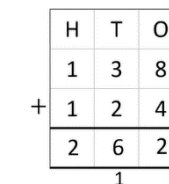
$$222 + 60 = \square$$

$$222 + 600 = \square$$

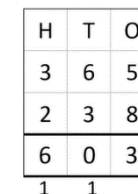
No regrouping



Regrouping



Regrouping ones and tens



Mathematical Talk

Addition is commutative and can be done in any order.

The sum of __ ones and __ ones is __ ones, this can be regrouped into __ tens and __ ones

The sum of __ tens and __ tens is __ tens, this can be regrouped into __ hundreds and __ tens

The sum of __ hundreds and __ hundreds is __ hundreds

And __ hundreds, __ tens and __ ones are equal to __

Ready to progress expectation

Secure fluency in addition and subtraction facts that bridge 10, through continued practice

Calculate complements to 100

Add and subtract up to three-digit numbers using columnar methods.

Manipulate the additive relationship: Understand the inverse relationship between addition and subtraction, and how both relate to the part-part-whole structure. Understand and use the commutative property of addition, and understand the related property for subtraction

Year 4

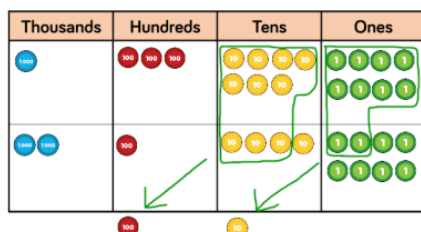
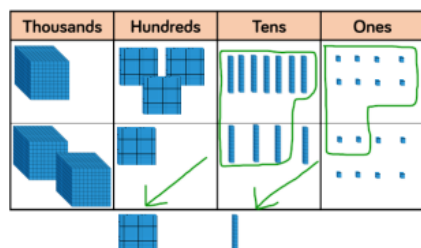
Addition

National Curriculum

- Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.
- Estimate and use inverse operations to check answers to a calculation
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
- Solve simple measure and money problems involving fractions and decimals to 2 decimal places
- Estimate, compare and calculate different measures, including money in pounds and pence

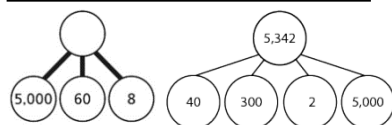
Concrete

Continue to make links to place value at each stage

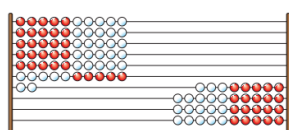
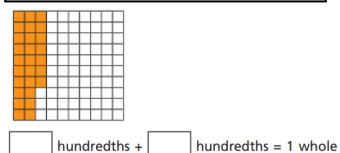


Pictorial

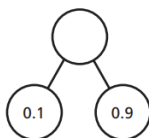
Variation in presentation of the units



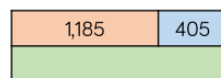
Conceptual Variation - decimals



$$\square + \square = 1$$



Representing



Money – expanded method



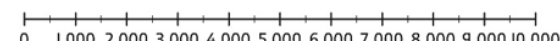
$$£3 + £1 = £4$$

$$50p + 23p = 73p$$

Abstract

Use rounding and estimating on a number line

$$912 + 6,149 = ?$$



$$1,000 + 6,000 = 7,000.$$

4-digit + 3-digit etc

	Th	H	T	O
	4	8	2	6
+		1	7	8
	5	0	0	4
	1	1	1	

More than two numbers

$$\begin{array}{r} 1,649 \\ 3,104 \\ + 516 \\ \hline 5,269 \\ 1 \quad 1 \end{array}$$

Regroup

	Th	H	T	O
	1	5	5	4
+	4	2	3	7
				1

Adding money

$$\begin{array}{r} 4.42 \\ + 7.63 \\ \hline \end{array}$$

Regrouping

	1	3	7	8
+	2	1	4	8
	3	5	2	6
	1	1		

Missing number problems

	Th	H	T	O
	4		6	
+	2	5		1
		7	8	9

Mathematical Talk

Addition is commutative and can be done in any order.

The sum of ones and ones is ones, this can be regrouped into tens and ones

The sum of tens and tens is tens, this can be regrouped into hundreds and tens

The sum of hundreds and hundreds is hundreds, this can be regrouped into thousands and hundreds

And thousands, hundreds, tens and ones are equal to

How many tenths make a whole? How many hundredths make a whole?

The sum of tenths and tenths is tenths. This can be regrouped into whole and tenths, which equals

Ready to progress expectation

No addition expectations

Number and place value

Recognise the place value of each digit in four-digit numbers and compose and decompose four-digit numbers using standard and non-standard partitioning.

Reason about the location of any four-digit number in the linear number system, including identifying the previous and next multiple of 1,000 and 100, and rounding to the nearest of each

Year 5

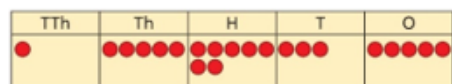
Addition

National Curriculum

- Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- Add and subtract numbers mentally with increasingly large numbers
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving number up to 3 decimal places.

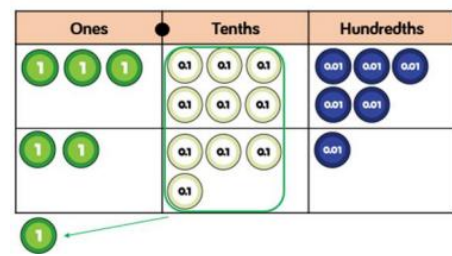
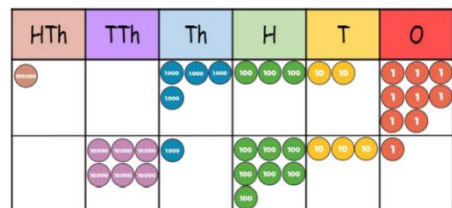
Concrete

Unitising



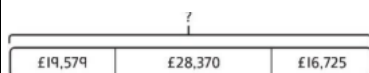
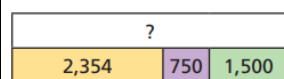
+ 4,012

Place value counters – build it

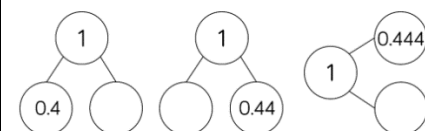


Pictorial

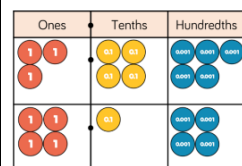
Representing



Complements to one whole using bonds

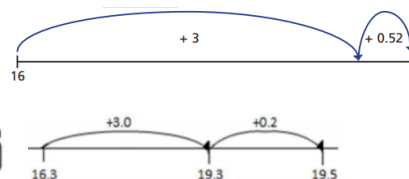


Alongside abstract



$$\begin{array}{r} 3.45 \\ + 4.14 \\ \hline \end{array}$$

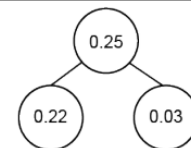
Partition



Bridging

$$\begin{array}{l} 0.64 + 0.59 = \\ 0.64 + 0.36 + 0.23 = \\ 1 + 0.23 = 1.23 \end{array}$$

Partition – non-standard



Abstract – at this stage children should be encouraged to work in the abstract to add larger numbers efficiently

Scaling facts by one-tenth or one-hundredth

$$\begin{array}{l} 8 + 6 = 14 \\ 0.8 + 0.6 = 1.4 \\ 0.08 + 0.06 = 0.14 \end{array}$$

Column addition

$$\begin{array}{r} \text{TTh Th H T O} \\ 23405 \\ + 7892 \\ \hline 31297 \end{array}$$

$$\begin{array}{r} 104328 \\ + 61731 \\ \hline 166059 \end{array}$$

Use 23,000+8,000 to check

Using place holders to support lining up

$$\begin{array}{r} \text{O Tth Hth} \\ 3.40 \\ + 0.65 \\ \hline 4.05 \end{array}$$

$$\begin{array}{r} 274.1 \\ + 195.8 \\ \hline 469.9 \end{array}$$

Mathematical Talk

Addition is commutative and can be done in any order.

The Sum of __ thousandths and __ thousandths is __ thousandths. This can be regrouped into __ hundredths and __ thousandths.

The sum of __ hundredths and __ hundredths is __ hundredths. This can be regrouped into __ tenths and __ hundredths.

The sum of __ tenths and __ tenths is __ tenths. This can be regrouped into __ whole and __ tenths.

__ whole ones __ tenths __ hundredths and __ thousandths is (read each digit)

Examples –

2 whole ones, 3 tenths, 4 hundredths and 6 thousandths is equal to two point three four six

18 hundredths are equal to 1 tenth and 8 hundredths, which is 0.18 (nought/zero point one eight)

Ready to progress expectation

No addition expectation

Apply place-value knowledge to known additive number facts (scaling facts by 1 tenth or 1 hundredth).

Recognise the place value of each digit in numbers with up to 2 decimal places, and compose and decompose numbers with up to 2 decimal places using standard and non-standard partitioning.

Year 6 Addition

National Curriculum

- Perform mental calculations, including with mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations involving the 4 operations
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Concrete

Represent 7-digit numbers



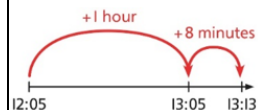
$$2,411,301 + 500,000 = ?$$

Continue to challenge place value

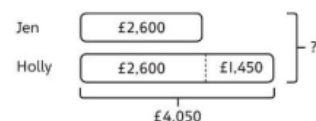
1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Pictorial

Number lines in problem solving and measure contexts



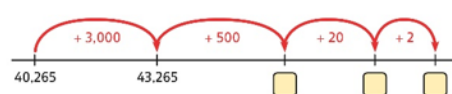
Representing multi-step



Encouraging mental strategies

- $3.8 + 2.4 = 4 + 2.2$
- $6.9 + 3.7 = 7 + 3.7 - 0.1$
- $1.7 + 1.8 = 1.7 + 1.7 + 0.1$

Choose most efficient methods



	TTh	Th	H	T	O
	4	0	2	6	5
+	3	5	2	2	

Abstract

Place value calculations

$$4,000 + 30,000 + 0.2 + 5,000,000 = \boxed{}$$

Adding in the most efficient way

$$\begin{array}{r} 1,649 \\ 3,104 \\ + 516 \\ \hline 5,269 \\ \hline 1 \quad 1 \end{array}$$

Recognise inverses

$$530,542 - 346,221 = 184,321$$

Missing number

	2	1		8	5
+			0	6	
	4	1	0		2

Continue with place holders

2	3	.	3	6	1
9	.	0	8	0	
5	9	.	7	7	0
+	1	.	3	0	0
9	3	.	5	1	1
2	1	.	2		

Mathematical Talk

Addition is commutative and can be done in any order.
All the above

Ready to progress expectation

Understand that 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number).

Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding.

Solve problems with 2 unknowns.

Subtraction

Year 1

Subtraction

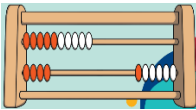
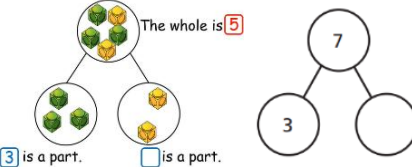
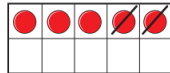
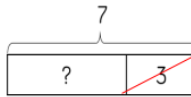
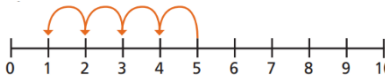
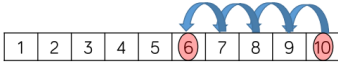
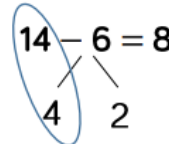
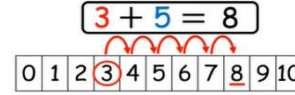
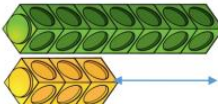
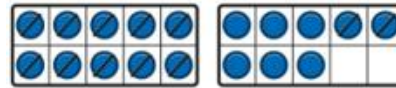
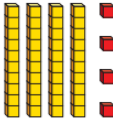
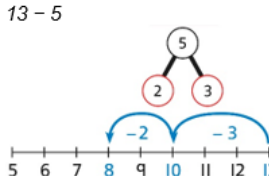
National Curriculum

Given a number, identify one less.

Subtract one-digit and two-digit numbers to 20, including zero.

Represent and use number bonds and related subtraction facts within 20.

Solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = _ - 9$.

Concrete	Pictorial (representations)	Abstract	
 Rekenreks.	Break apart 	Draw the dots and cross them out. 	Equals at the start $5 - 3 = \square$ $\square = 6 - 1$
 Numicon and number shapes for counting, ordering, patterns, number bonds and calculating. 10 take away 3	Representing  Find the difference. 	Number line – count back 	Comparing number sentences $12 + 3 \bigcirc 12 - 3$
$10 - 4 = 6$  Number track- children find the minuend and count back using a counter	Partition and count back through 10. 	Number line -counting on. $3 + 5 = 8$ 	Inverses $\square - 5 = 4$
 Cubes – finding the difference	Subtracting 10s and 1s  First subtract the 10, then subtract 2.	Use empty box problems $3 + \square = 8$ $3 + \square = 9$ $3 + \square = 10$	
 Dienes to support understanding of place value and number size.	$14 - 6 = 8$ 		

Mathematical Talk

Subtraction is not commutative

This is the whole, this is part of the whole

__ones take away __ones equal __ones

One ten is equal to 10 ones

Ready to progress expectation

Count within 100, forwards and backwards, starting with any number

Compose numbers to 10 from 2 parts, and partition numbers to 10 into parts, including recognising odd and even numbers.

Reason about the location of numbers to 20 within the linear number system, including comparing using $<$ $>$ and $=$

Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.

Develop fluency in addition and subtraction facts within 10

Year 2

Subtraction

National Curriculum

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

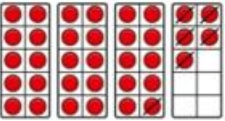
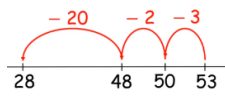
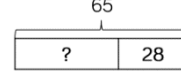
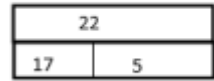
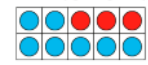
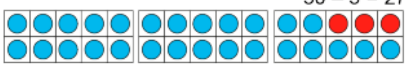
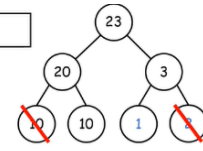
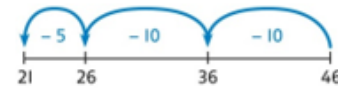
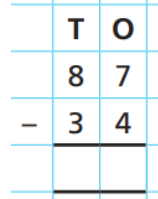
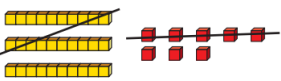
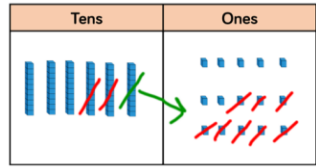
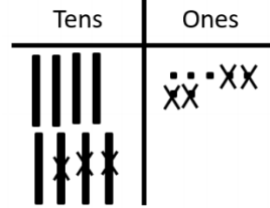
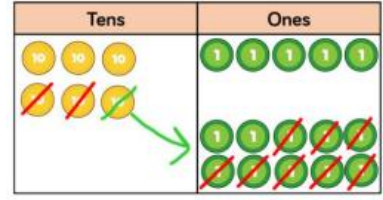
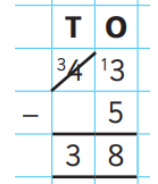
- a two-digit number and ones
- a two-digit number and tens
- two, two-digit numbers

Solve problems with addition and subtraction

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100

Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Concrete	Pictorial (representations)	Abstract
Taking away (-5 then -1) 	Crossing 10 recap.  Representing 	Explore related facts $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$ 
Practise exchanging  Using known facts $10 - 3 = 7$ $30 - 3 = 27$  	Partition parts $23 - 12 = \square$  Number line 10s and 1s 	No exchange Partition $80 + 7$ $- 30 + 4$ $50 + 3$ Column subtraction 
No exchange  Exchange  $\begin{array}{r} 5 \text{ } 1 \\ 65 \\ - 28 \\ \hline 37 \end{array}$	Scaffold by drawing sticks and dots or place value counters. No exchange  Exchanging 	Exchanging $\begin{array}{r} 60 \quad 13 \\ 70 + 3 \\ - 40 + 6 \\ \hline 20 + 7 \end{array}$  

Mathematical Talk

Subtraction is not commutative

When we take away tens the ones digit stays the same.

___ones take away ___one is equal to ___ones, ___tens take away ___tens is equal to ___tens

So ___ - ___ is equal to ___tens and ___ones which is equal to ___

I can see there aren't enough ones for me to take away ___ones without exchanging

So we can exchange one ten for ten ones

Now ___ones take away ___ones equal ___ones, ___tens take away ___tens equal ___tens

So ___ - ___ is equal to ___tens and ___ones which is equal to ___

Ready to progress expectation

Add and subtract across 10

Recognise the subtraction structure of 'difference' and answer questions of the form, "How many more...?"

Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract only ones or only tens to/from a two-digit number

Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract any 2 two-digit numbers.

Year 3

Subtraction

National Curriculum

Add and subtract numbers mentally, including:

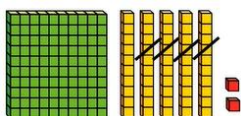
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Concrete

No exchange



Exchanging

$$434 - 72$$

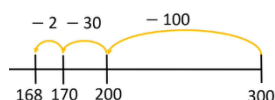
H	T	O
400	30	4
300	20	4

$$435 - 273 = 262$$

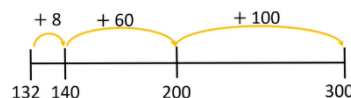
Hundreds	Tens	Ones
400	30	5
200	70	3

Pictorial

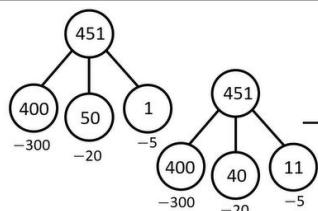
Number line - partition



Checking answers counting on



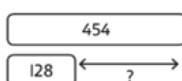
Part whole showing exchange



Exchanging

Hundreds	Tens	Ones
400	50	1
300	20	0

Representing



Related facts

247
173 74

$$173 + 74 = 247$$

$$74 + 173 = 247$$

$$247 - 173 = 74$$

$$247 - 74 = 173$$

$$323 - 141$$

Hundreds	Tens	Ones
300	20	3
100	40	1

Abstract

Variation –
What's the same?
What's different?

236	436
-----	-----

Use scaling

$$14 - 6 = 8$$

$$140 - 60 = 80$$

It's important that children understand during the exchange that 4 hundreds have been regrouped into 3 hundreds and 10 tens.

H	T	O
4	3	4
3	7	2

More than one exchange – refer to mathematical talk.

H	T	O
4	3	4
3	7	2

Exchanging a hundred and then exchanging a ten

H	T	O
4	3	4
3	7	2

Reasoning – what has gone wrong?

H	T	O
4	3	4
3	7	2

Mathematical Talk

Subtraction is not commutative; it cannot be done in any order.

ones take away ones equal ones, tens take away tens equal tens and hundreds take away hundreds equal hundreds so take away is equal to hundreds tens and ones which equals

ones take away ones equal ones. I can see there aren't enough tens for me to take away tens without exchanging so we can exchange one hundred for ten tens

Now tens take away tens equal tens, hundreds take away hundreds equal hundreds

So - is equal to hundreds tens and ones which is equal to

Ready to progress expectation

Secure fluency in addition and subtraction facts that bridge 10, through continued practice

Calculate complements to 100

Add and subtract up to three-digit numbers using columnar methods.

Manipulate the additive relationship: Understand the inverse relationship between addition and subtraction, and how both relate to the part-part-whole structure. Understand and use the commutative property of addition, and understand the related property for subtraction

Year 4

Subtraction

National Curriculum

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.

Estimate and use inverse operations to check answers to a calculation

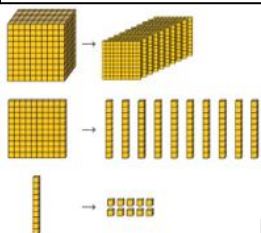
Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

Solve simple measure and money problems involving fractions and decimals to 2 decimal places

Estimate, compare and calculate different measures, including money in pounds and pence

Concrete

Continue to recognise exchanges



No exchange

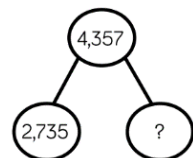
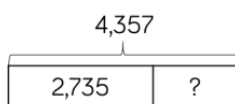
Th	H	T	O

Exchanging

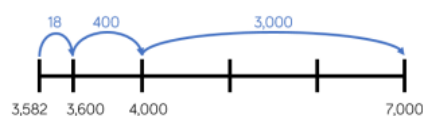
Thousands	Hundreds	Tens	Ones

Pictorial

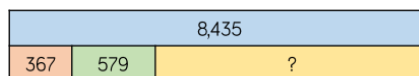
Representing



Mental strategies



Two-step representation

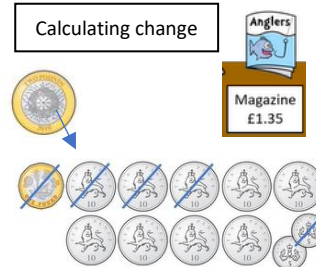


Alongside abstract

Th	H	T	O

Th	H	T	O
3	10	5	15
2	1	2	8
1	9	3	7

Calculating change



Abstract

Subtrahend has fewer digits

$$\begin{array}{r} 2,796 \\ - 485 \\ \hline 2,311 \end{array}$$

With exchanges

$$\begin{array}{r} 514121 \\ 8,538 \\ - 2,789 \\ \hline 3,749 \end{array}$$

Exchange through zero

$$\begin{array}{r} 391 \\ 8,403 \\ - 2,176 \\ \hline 6,227 \end{array}$$

Subtraction in context

$$\begin{array}{r} 514121 \\ 6532 \\ - 3796 \\ \hline £2736 \end{array}$$

Checking strategies

$$1,319 + 4,484 = 5,803$$

$$5,803 - 4,484 = 1,319$$

Th	H	T	O
5	7	19	13
4	4	8	4
1	3	1	9

Mathematical Talk

Subtraction is not commutative; it cannot be done in any order.

_ones take away _ones equal _ones, _tens take away _tens equal _tens, _hundreds take away _hundreds equal _hundreds, _thousands take away _thousands equal _thousands so __ take away __ is equal to _thousands _hundreds _tens and _ones which equals __

I can see there aren't enough tens for me to take away __ tens without exchanging but we have zero hundreds so we must exchange one thousand for ten hundreds, now I can exchange one hundred for ten tens. Now __ tens take away __ tens equals __ tens etc.

Ready to progress expectation

No subtraction expectation

Number Facts

Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100)

Recognise the place value of each digit in four-digit numbers and compose and decompose four-digit numbers using standard and non-standard partitioning.

Year 5

Subtraction

National Curriculum

Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

Add and subtract numbers mentally with increasingly large numbers

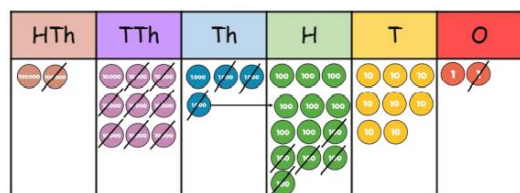
Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

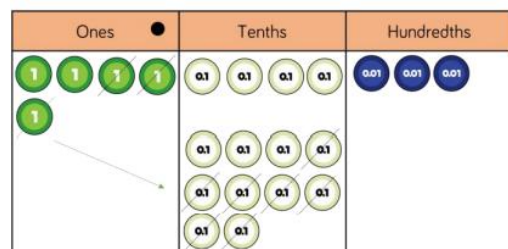
Solve problems involving number up to 3 decimal places.

Concrete

$$294,382 - 182,501 = 111,881$$



$$5.43 - 2.7 = 2.73$$



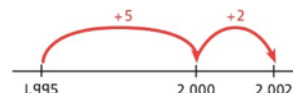
Pictorial - Use number lines when working with time and temperature (negative numbers)

Continue to encourage mental strategies

$$827 - 99 = 827 - 100 + 1 = 728$$

Use efficient methods

$$2,002 - 1,995 = ?$$



$$0.42 - 0.3 =$$



Multi-step

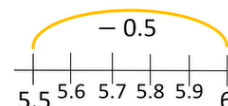
23,000	
12,365	4,017

Alongside abstract



Number lines

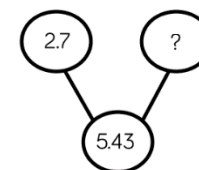
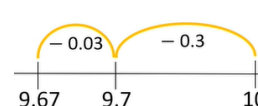
Count back



Representing

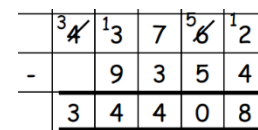


Partition

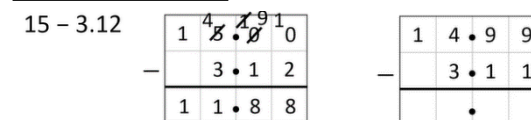


Abstract - At this stage children should be encouraged to work in the abstract to subtract larger numbers efficiently

Multiple exchanges

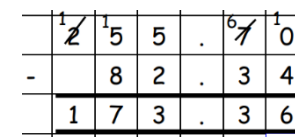


Constant difference



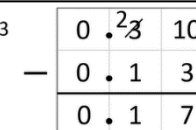
Place holders and decimal point in line

$$225.7 - 82.34 =$$



Continue to develop strong understanding of place value

What is the difference between 13 hundredths and 3 tenths?



Mathematical Talk

Subtraction is not commutative; it cannot be done in any order.

__ hundredths take away __ hundredths equal __ hundredths, __ tenths take away __ tenths equal __ tenths, __ whole ones take away __ whole ones equal __ whole ones and __ whole ones tenths and hundredths equals (read each digit).

I can see there aren't enough hundredths to take away __ hundredths so I need to exchange one tenth for 10 hundredths, now __ hundredths take away __ hundredths equal __ hundredths etc

I can see there aren't enough hundredths to take away __ hundredths but I don't have any tenths to allow for an exchange so I need to exchange 1 whole for 10 tenths now I can exchange 1 tenth for 10 hundredths etc

Ready to progress expectation

No subtraction expectation

Recognise the place value of each digit in numbers with up to 2 decimal places, and compose and decompose numbers with up to 2 decimal places using standard and nonstandard partitioning.

Reason about the location of any number with up to 2 decimal places in the linear number system, including identifying the previous and next multiple of 1 and 0.1 and rounding to the nearest of each.

Year 6

Subtraction

National Curriculum

Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

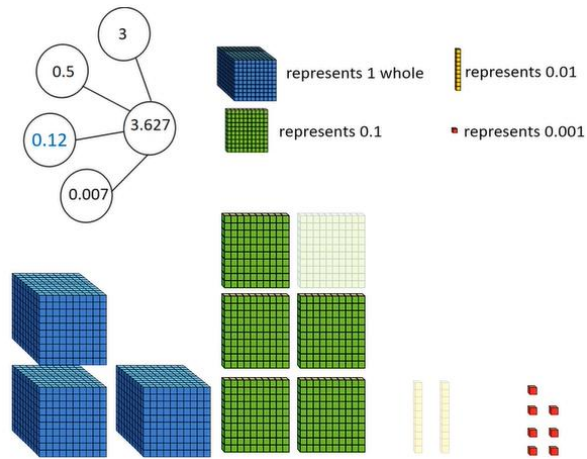
Add and subtract numbers mentally with increasingly large numbers

Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

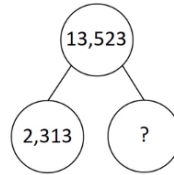
Concrete (As year 5)

Continue to visualise with base 10 and partition in different ways



Pictorial (As year 5)

Representing



Recognise inverses

$$\square - 346,221 = 184,321$$

?
346,221 184,321

Related facts

3,735
3,612 123

$$\begin{aligned} 3,612 + 123 &= 3,735 \\ 123 + 3,612 &= 3,735 \\ 3,735 - 123 &= 3,612 \\ 3,735 - 3,612 &= 123 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{Part + part = whole} \\ \\ \text{Whole - part = part} \end{array}$$

Abstract - At this stage children should be encouraged to work in the abstract to subtract larger numbers efficiently

Numbers to 10 million

	2	9	3	¹ 3	8	2
-	1	8	2	5	0	1
	1	1	1	8	8	1

Choose appropriate methods

$$\begin{aligned} &834,259 - 299,999 \\ &= 834,259 - 300,000 + 1 \end{aligned}$$

Lining up decimals and using place holders

$$\begin{aligned} &225.7 - 82.34 = \\ &\begin{array}{r} \overset{1}{\cancel{2}} \overset{1}{5} 5 . \overset{6}{\cancel{7}} \overset{1}{0} \\ - 8 2 . 3 4 \\ \hline 1 7 3 . 3 6 \end{array} \end{aligned}$$

Strategies for missing number problems

TTh	Th	H	T	O
6		1	3	
-	1	4		6
	8	1	1	5

? - 6 ones = 5 ones

TTh	Th	H	T	O
5	1	2	1	1
-	1	4	0	1
	4	8	1	5

12 thousand

8 Th	4 Th
------	------

Including in context

1	0	5	.	4	1	9	kg
-		3	6	.	0	8	kg
		6	9	.	3	3	kg

Mathematical Talk

Subtraction is not commutative; it cannot be done in any order.

__hundredths take away __ hundredths equal __hundredths, __tenths take away __tenths equal __tenths, __whole ones take away __whole ones equal __ whole ones and __whole ones __tenths and __hundredths equals (read each digit).

I can see there aren't enough hundredths to take away __hundredths so I need to exchange one tenth for 10 hundredths, now __hundredths take away __hundredths equal __hundredths etc

I can see there aren't enough hundredths to take away __hundredths but I don't have any tenths to allow for an exchange so I need to exchange 1 whole for 10 tenths now I can exchange 1 tenth for 10 hundredths etc

Ready to progress expectation

Understand that 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number).

Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding.

Solve problems with 2 unknowns.

Multiplication

Year 1

Multiplication

National Curriculum

solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

Create equal groups

Represent on the hundred square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

There are 4 equal groups of 5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

5 + 5 + 5 + 5 = 20

Skip counting with coins.

5	10	15	20	25	30	35	40	45	50	55	60
---	----	----	----	----	----	----	----	----	----	----	----

Support with number track.

Create arrays.

Showing equal groups on the number line. Start skip counting.

Draw dots in circles to show 3 equal groups of 5

Draw the array

Bar model to represent a problem

 × =

Recognise as repeat addition

 + + + = 20

Introduce the multiplication symbol

There are 3 equal groups of 4

4 + 4 + 4 = 12

3 × 4 = 12

Empty box problems - variation

3 × □ + 2 = 20

3 × □ + 2 = 23

3 × □ + 2 = 26

Mathematical Talk

There are groups and in each group

There are equal groups of

Let's skip count in 2s

Arrays - There are rows of , there are columns of

So into our repeat addition we have a , another , and another , so we have 3 equal groups of

4 times 2, we can read this as 4 lots of 2

Multiplication is commutative and can be done in any order

Ready to progress expectation

Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count forwards and backwards through the odd numbers.

Year 2

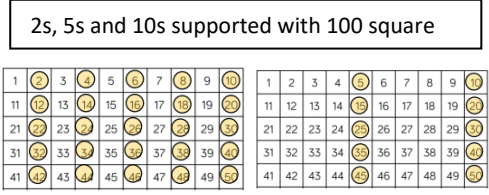
Multiplication

National Curriculum

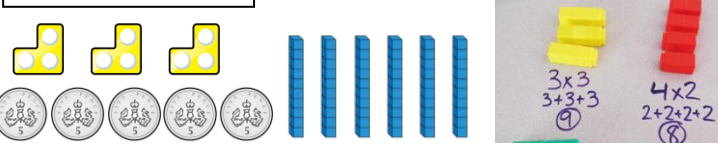
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the (×), (÷) and equals (=) signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.

Concrete

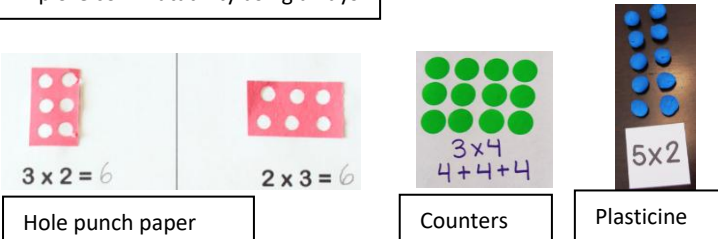
2s, 5s and 10s supported with 100 square



Make equal groups

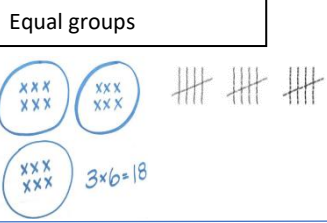


Explore commutativity using arrays

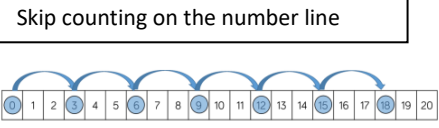


Pictorial (representations)

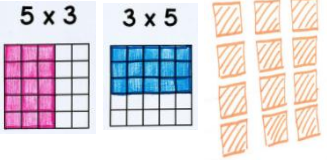
Equal groups



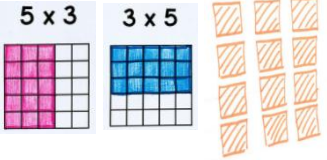
Skip counting on the number line



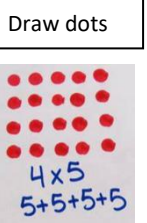
Arrays



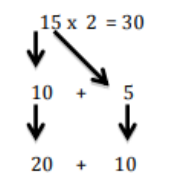
Squares



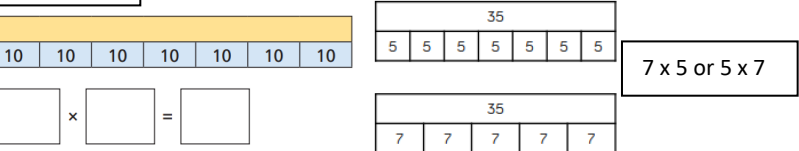
Draw dots



Doubling 2-digit



Bar models



Abstract

Using the multiplication symbol

Picture	Multiplication	Sentence
	4 × 10 = 40	4 lots of 10 is equal to 40

5 × 3 = 5 + 5 + 5 =

lots of =

× =

Commutative number sentences

+ =

× =

+ =

× =



Mathematical Talk

There are equal groups with in each group
 add add add so we have equal groups of
What is the same and what is different about the bar models?
So rows of is equal to rows of
Multiplication can be done in any order.

Ready to progress expectation

Recognise repeated addition contexts, representing them with multiplication equations and calculating the product, within the 2, 5 and 10 multiplication tables.

Relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor, and to division equations.

Year 4

Multiplication

National Curriculum

Recall multiplication and division facts for multiplication tables up to 12×12

Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers

Recognise and use factor pairs and commutativity in mental calculations

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Concrete

Build it

Tens	Ones

Hundreds	Tens	Ones

	H	T	O
	2	4	5
$\times$			4

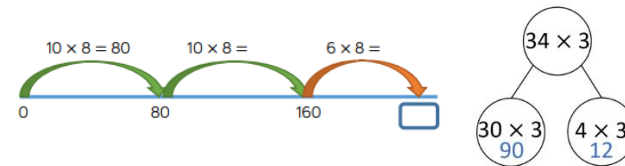
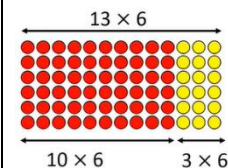
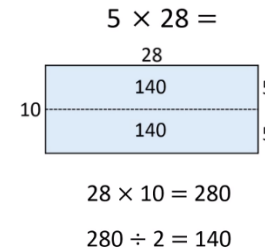
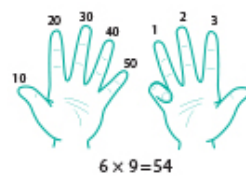
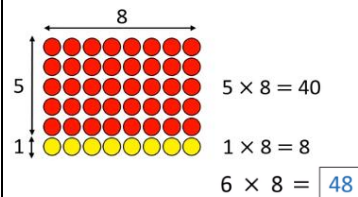
Overcome misconceptions – zero tens

Hundreds	Tens	Ones

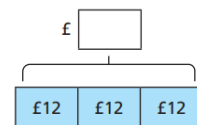
	H	T	O
	2	0	3
$\times$			3

Pictorial

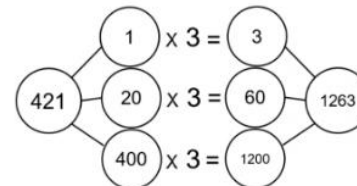
Efficient methods



Representing



Partition – distributive law



Abstract

Explore commutativity

$$2 \times 5 \times 3 \quad 3 \times 2 \times 5 \quad 5 \times 3 \times 2$$

$$10 \times 3 \quad 6 \times 5 \quad 15 \times 2$$

Mental strategies

£3.25	£3.25	£3.25
-------	-------	-------

$$£3 \times 3 = £9$$

$$25p \times 3 = 75p$$

Expanded

	H	T	O
$\times$	1	2	3
			4
		1	2
		8	0
$+$	4	0	0
	4	9	2

$$4 \times 7 = 28$$

$$4 \times 70 = 280$$

$$40 \times 7 = 280$$

$$4 \times 700 = 2,800$$

$$400 \times 7 = 2,800$$

Column method

	3	4	2
$\times$			7
	2	3	9
	2	1	

Mathematical Talk

Multiplication is commutative and can be done in any order

_lots of _ones is _ones (Do you need to regroup into _tens and _ones?)

_lots of _tens is _tens (Do you need to regroup into _hundreds and tens?)

_lots of _hundreds is _hundreds (Do you need to regroup into _thousands and _hundreds?)

The product of _ and _ is _

Ready to progress expectation

Understand and apply the distributive property of multiplication

Manipulate multiplication and division equations, and understand and apply the commutative property of multiplication.

Multiply and divide whole numbers by 10 and 100 (keeping to whole number quotients); understand this as equivalent to making a number 10 or 100 times the size.

Year 5

Multiplication

National Curriculum

Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

Multiply and divide numbers mentally drawing upon known facts

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Concrete

Build it

Th	H	T	O
1,000 1,000	100 100	10	1 1
1,000 1,000	100 100	10	1 1
1,000 1,000	100 100	10	1 1
1,000 1,000	100 100	10	1 1

x	30	2
10	300 300	20 20
4	400 300 200	8 8 8 8

Pictorial

Grid to support multiplying by powers of 10

2-digit x 2-digit

x	30	4
20	600	80
3	90	12

$$600 + 90 + 80 + 12 = 782$$

3-digit x 2-digit

x	200	30	4
30	6,000	900	120
2	400	60	8

4-digit x 2-digit

x	2,000	300	10	3
30	60,000	9,000	300	90
2	4,000	600	20	6

$$60,000 + 9,000 + 4,000 + 600 + 300 + 90 + 20 + 6 =$$

74,016

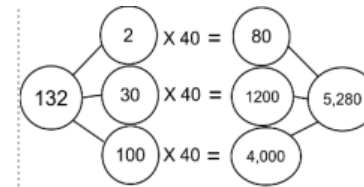
Multiplication grid for support when focusing upon use of the

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Representing

814
814 814 814 814

Partition – distributive law



Abstract -encourage to estimate first

4-digit x 1-digit

	Th	H	T	O
	1	8	2	6
x				3
	5	4	7	8

2 1

2-digit x 2-digit

	H	T	O
	4	1	
x	2	6	
	8	2	0
+	8	2	0
1	0	6	6

(41 × 6)

(41 × 20)

3-digit x 2-digit

	Th	H	T	O
	3	2	6	
x		3	2	
	6	5	2	
+	9	7	8	0
1	0	4	3	2

(326 × 2)

(326 × 30)

4-digit x 2-digit

	TTh	Th	H	T	O
		2	7	3	9
x				2	8
	2	1	9	1	2
+	5	4	7	8	0
1	7	6	6	9	2

1

Mathematical Talk

Multiplication is commutative and can be done in any order

Same as year 4 when multiplying by one-digit with the extra thousands digit to consider.

First consider the ones as the multiplier. Then consider the tens as the multiplier.

Discuss needing a place holder when using the tens as a multiplier.

Discuss the value of each digit as you move through each part of the long multiplication method but use known facts to support..

Ready to progress expectation

Secure fluency in multiplication table facts, and corresponding division facts, through continued practice.

Multiply and divide numbers by 10 and 100; understand this as equivalent to making a number 10 or 100 times the size, or 1 tenth or 1 hundredth times the size.

Find factors and multiples of positive whole numbers, including common factors and common multiples, and express a given number as a product of 2 or 3 factors.

Multiply any whole number with up to 4 digits by any one-digit number using a formal written method.

Divide a number with up to 4 digits by a one-digit number using a formal written method, and interpret remainders appropriately for the context.

Year 6 Multiplication

National Curriculum

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- Perform mental calculations, including with mixed operations and large numbers
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- Multiply one-digit numbers with up to 2 decimal places by whole numbers

Concrete

Know each power of 10 is equal to 1 group of 10 of the next smallest power of 10. Eg. 1 million is equal to 10 hundred thousands.



Place value charts to understand the movement of the digits.

	Th	H	T	O	.	Tth
$2.5 \times 10 = 25$			2	5		
$2.5 \times 100 = 250$		2	5			
$2.5 \times 1,000 = 2,500$	2	5	0	0		

	H	T	O	.	Tth	Hth
2×3			6	.		
0.2×3			0	.	6	
0.02×3				.		

Pictorial

Distributive law

$$124 \times 26 = 124 \times 20 + 124 \times 6$$

$$= 124 \times 2 \times 10 + 124 \times 6$$

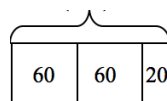
$$= 2,480 + 744$$

$$= 3,224$$

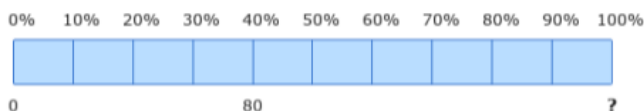
Compensation

$$5 \times 8 = 10 \times 4 \quad 0.3 \times 320 = 3 \times ?$$

Representing



60mph for $2\frac{1}{3}$ hours



Abstract – Same as year 5

Up to 4-digit
x 2-digit

	TTh	Th	H	T	O
		1	2	0	7
$\times$				3	6
		7	2	4	2
$+$	3	6	2	1	0
	4	3	4	5	2

(1,207 $\times$ 6)
(1,207 $\times$ 30)

Use known facts

$$4 \times 3 = 12$$

$$4 \times 0.3 = 1.2$$

$$4 \times 0.03 = 0.12$$

$$3 \times 4 = 12$$

$$3 \times 40,000 = 120,000$$

$$300 \times 400 = 120,000$$

$$20 \times 5 = 100$$

$$20 \times 0.5 = 10$$

$$20 \times 0.05 = 1$$

Explore efficient methods

36% of 450 =

$$10\% \text{ of } 450 = 45 \quad 1\% \text{ of } 450 = 4.5$$

$$30\% \text{ of } 450 = 135 \quad 6\% \text{ of } 450 = 27$$

$$135 + 27 = 162$$

Multiply decimals by integers

2	1	.	3
$\times$			4
8	5	.	2

1

2	.	4	7
$\times$			6
1	4	.	8
	2		4

Multiplying without the decimal and reinsert

$$256 \times 8.5 = \square$$

$$256 \times 85 \div 10$$

$$36 \times 4.5 = 4.5 \times 36$$

$$= 36 \times 4 + 18$$

or

$$36 \times 4.5 = 36 \times 45 \div 10$$

Mathematical Talk

Multiplication is commutative and can be done in any order

The same as previous years when multiplying by one digit and two-digit numbers.

_lots of _hundredths is _hundredths (Do I need to regroup?)

_lots of _tenths is _tenths (Do I need to regroup?)

Ready to progress expectation

Understand that 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number).

Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding.

Solve problems involving ratio relationships.

Solve problems with 2 unknowns

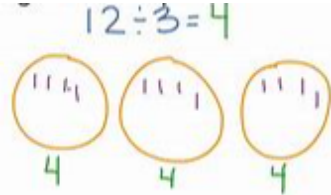
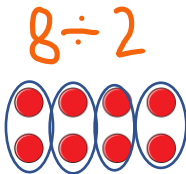
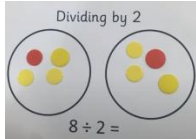
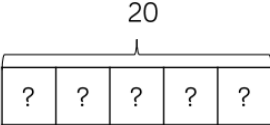
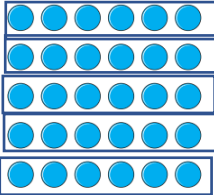
Division

Year 1

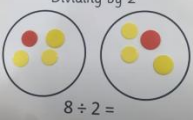
Division

National Curriculum

solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

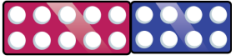
Concrete	Pictorial (representations)	Abstract
Using objects to share into groups Grouping objects into 2s SHARING Divide 12 into 2 equal groups.  There are 6 in each group. GROUPING Divide 12 into groups of 2.  There are 6 groups of 2.	Share into 3 groups  Circle groups of 2 	Using concrete, pictorial and teacher support
Dividing by 2  8 ÷ 2 = Sharing - Using counters to share into equal groups.  Grouping - Create the number then place the number shape they are dividing by over the top to see how many groups. 	Draw the number line and show the jumps back.  20 ÷ 4 = 5 Bar Models  Make connections with arrays 	
Count back on the number track. Repeat subtraction. Count the jumps. 		

Dividing by 2



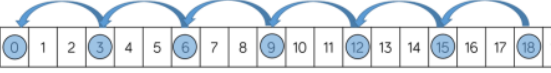
8 ÷ 2 =

Sharing - Using counters to share into equal groups.



Grouping - Create the number then place the number shape they are dividing by over the top to see how many groups.

Count back on the number track. Repeat subtraction. Count the jumps.



Mathematical Talk

There are __ groups
There are __ equal groups of __
I am sharing __ ones into __ equal groups
There are __ ones in each group.

Ready to progress expectation

Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count forwards and backwards through the odd numbers.

Year 2 Division

National Curriculum

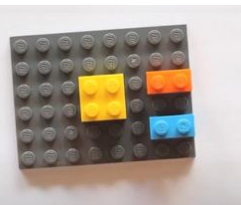
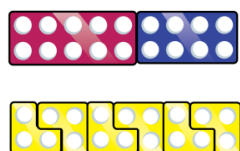
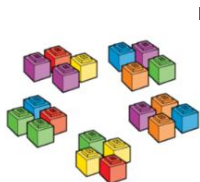
Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the ($\times$), ($\div$) and equals (=) signs.

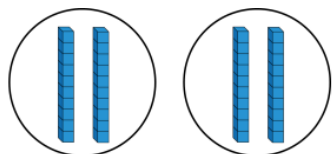
Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.

Concrete

Share and group



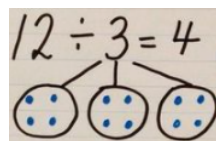
Lego – place smaller equal sized bricks over the top of the dividend (the larger brick).



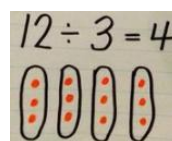
$$40 \div 2 = 20$$

Pictorial (representations)

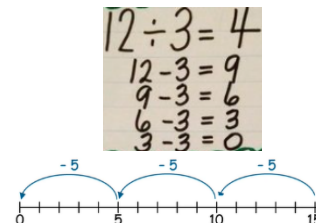
Sharing into equal groups



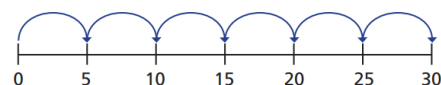
Grouping



Repeat subtraction

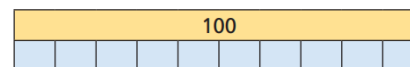


Skip counting using number line



30 is made of equal groups of

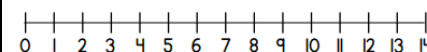
Bar models



Abstract

Using concrete and pictorial

Work out $14 \div 2 =$



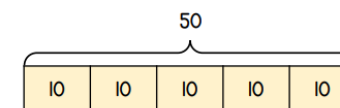
Dora has 10 sweets.



She shares them equally between 2 plates.



Here is a bar model.



Use it to complete the number sentences.

$$50 \div \square = 10 \quad 50 \div 10 = \square$$

$$35 \div 5 \bigcirc 40 \div 10$$

Mathematical Talk

I am sharing ones into equal groups

There are ones in each group.

I will circle equal groups of

I have made groups

There are equal groups of

If we skip count in we can see that is made up of equal groups of

Ready to progress expectation

Recognise repeated addition contexts, representing them with multiplication equations and calculating the product, within the 2, 5 and 10 multiplication tables.

Relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor, and to division equations.

Year 3

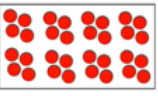
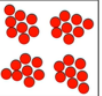
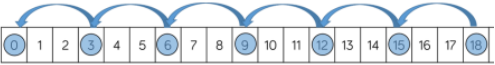
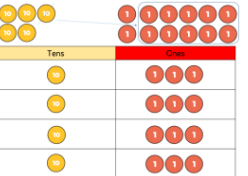
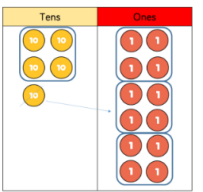
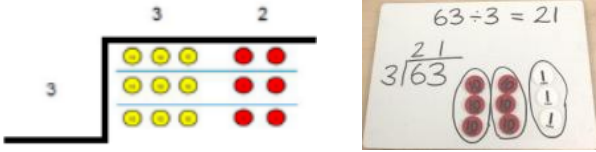
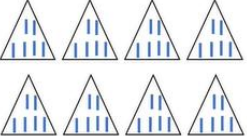
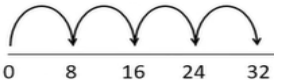
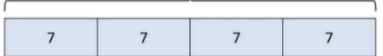
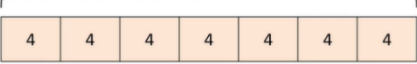
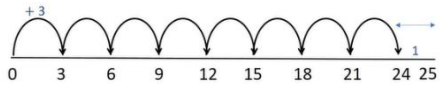
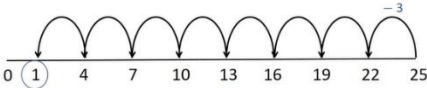
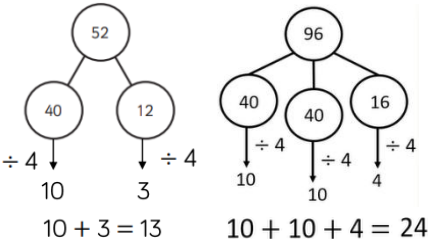
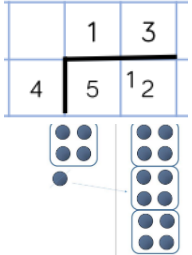
Division

National Curriculum

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Concrete	Pictorial	Abstract
Concrete $32 \div 8$ Shared into 8 equal groups  Grouped into 8s  Number track – counting back and forwards  Remainders $21 \div 4 = 5 \text{ r } 1$  Sharing (with exchange)  Grouping (with exchange)  Concrete alongside abstract 	Pictorial Sharing  Grouping  Shared into 4 equal parts  Grouped into 4s  Remainders $25 \div 3 =$   Flexible partitioning  Draw dots under the abstract 	Abstract Variation – What's the same? What's different? $14 \div 2 = \square$ $140 \div 2 = \square$ Building mental strategies $80 \xrightarrow{\div 2} 40 \xrightarrow{\div 2} 20 \xrightarrow{\div 2} 10$ Formal written method $\begin{array}{r} 32 \\ 3 \overline{) 96} \\ \underline{96} \\ 0 \end{array}$ $\begin{array}{r} 23 \text{ r } 3 \\ 4 \overline{) 95} \\ \underline{80} \\ 15 \end{array}$

Mathematical Talk

I am sharing ___ ones into ___ equal groups

There are ___ ones in each group.

I will circle equal groups of ___

I have made ___ groups

There are ___ equal groups of ___

If we skip count in ___ we can see that ___ is made up of ___ equal groups of ___

Can the remainder be more than the divisor?

What is the best way to partition this number so we can divide it into equal groups of ___?

Ready to progress expectation

Apply known multiplication and division facts to solve contextual problems with different structures, including quotitive and partitive division.

Year 4

Division

National Curriculum

Recall multiplication and division facts for multiplication tables up to 12×12

Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers

Recognise and use factor pairs and commutativity in mental calculations

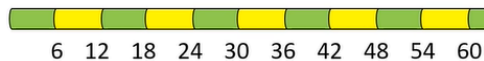
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Concrete

$\frac{\text{quotient}}{\text{divisor} \overline{) \text{dividend}}}$

Counting stick $42 \div 6 = ?$



Sharing – start with the largest place value

H	T	O
100 100	10	1
100 100	10	1
100 100	10	1
100 100	10	1

Place value chart to see the movement

Th	H	T	O
		● ● ● ●	● ● ● ●
		● ● ● ●	● ● ● ●

$\div 10$ $\div 10$ $\div 10$

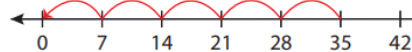
$$430 \div 10 = 43$$

Pictorial

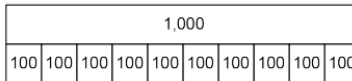
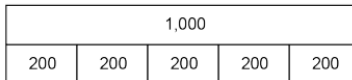
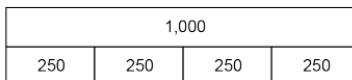
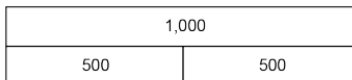
$\frac{\text{quotient}}{\text{divisor} \overline{) \text{dividend}}}$

Number lines

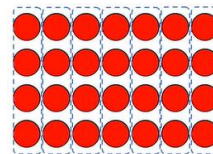
$$35 \div 7 = \underline{5}$$



Representing 1000 divided into 2,4,5 and 10 equal parts

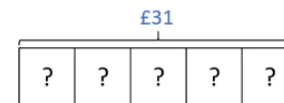


Fact families

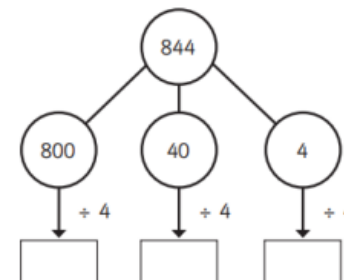
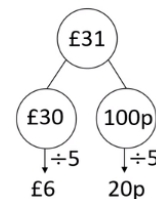


$$\begin{aligned}
 7 \times 4 &= 28 \\
 4 \times 7 &= 28 \\
 28 \div 4 &= 7 \\
 28 \div 7 &= 4
 \end{aligned}$$

Representing



Partitioning



Abstract -

$\frac{\text{quotient}}{\text{divisor} \overline{) \text{dividend}}}$

When to share and when to group?

$$1600 \div 2$$

$$200 \div 25$$

2 equal parts of 800

8 groups of 25 in 200?

Making connections

$$15 \div 3 = 5$$

$$1,500 \div 300 = 5$$

$$1,500 \div 3 = 500$$

Formal written method

$$\begin{array}{r}
 142 \\
 3 \overline{) 4126} \\
 \underline{3} \\
 11 \\
 \underline{9} \\
 21 \\
 \underline{21} \\
 0
 \end{array}$$

$$\begin{array}{r}
 195 \text{ r}3 \\
 4 \overline{) 7383} \\
 \underline{7} \\
 0 \\
 \underline{38} \\
 23 \\
 \underline{20} \\
 3
 \end{array}$$

Mathematical Talk

How many equal groups of _hundreds can I get out of _hundreds?

I have _hundreds remaining, I can regroup this into _tens.

Now how many equal groups of _tens can I get out of _tens?

I have _tens remaining which I can regroup into _ones.

How many equal groups of _ones can I get out of _ones?

Ready to progress expectation

Understand and apply the distributive property of multiplication

Manipulate multiplication and division equations, and understand and apply the commutative property of multiplication.

Multiply and divide whole numbers by 10 and 100 (keeping to whole number quotients); understand this as equivalent to making a number 10 or 100 times the size.

Year 5

Division

National Curriculum

Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

Multiply and divide numbers mentally drawing upon known facts

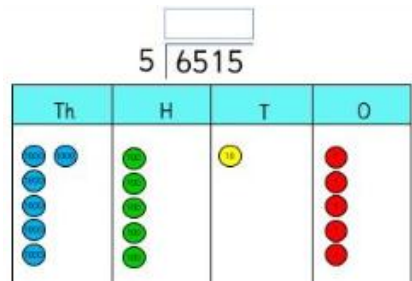
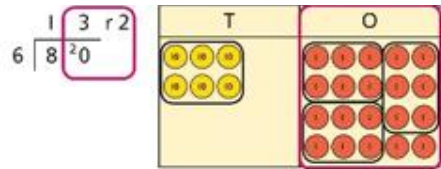
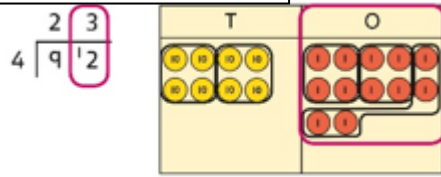
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Concrete

quotient
divisor)dividend

Create using place value counters



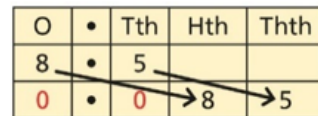
Pictorial – children can draw their own counters for a more pictorial method

See the movement of the digits

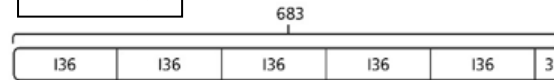


$$3,200 \div 100 = ?$$

Including decimals



Representing



$$683 \div 5 = 136 r 3$$

Use known facts

$$3,000 \div 5 = 600$$

$$3,000 \div 50 = 60$$

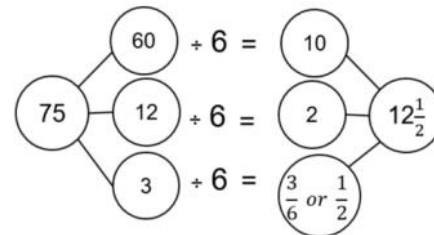
$$3,000 \div 500 = 6$$

$$5 \times 600 = 3,000$$

$$50 \times 60 = 3,000$$

$$500 \times 6 = 3,000$$

Partition



Abstract

quotient
divisor)dividend

Formal written method

$$\begin{array}{r} 0556 \\ 7 \overline{) 3383942} \end{array}$$

Use multiplication to check
 $556 \times 7 = ?$

Remainder

$$\begin{array}{r} 86r2 \\ 5 \overline{) 432} \end{array}$$

Fraction remainder

$$\begin{array}{r} 12\frac{1}{2} \\ 6 \overline{) 715} \end{array}$$

Decimal remainder

$$\begin{array}{r} 12.5 \\ 6 \overline{) 715.0} \end{array}$$

Mathematical Talk

How many equal groups of _ thousands can I get out of _ thousands?

I have _ thousands remaining. I can regroup this into _ hundreds.

How many equal groups of _ hundreds can I get out of _ hundreds?

I have _ hundreds remaining, I can regroup this into _ tens.

Now how many equal groups of _ tens can I get out of _ tens?

I have _ tens remaining which I can regroup into _ ones.

How many equal groups of _ ones can I get out of _ ones?

When we divide by 10 all the digits move one place the right. The value of each digit becomes 10 x smaller.

Ready to progress expectation

Secure fluency in multiplication table facts, and corresponding division facts, through continued practice.

Multiply and divide numbers by 10 and 100; understand this as equivalent to making a number 10 or 100 times the size, or 1 tenth or 1 hundredth times the size.

Find factors and multiples of positive whole numbers, including common factors and common multiples, and express a given number as a product of 2 or 3 factors.

Multiply any whole number with up to 4 digits by any one-digit number using a formal written method.

Year 6

Division

National Curriculum

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

Perform mental calculations, including with mixed operations and large numbers.

Solve problems involving addition, subtraction, multiplication and division

Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Concrete

quotient
divisor) dividend

Counters underneath

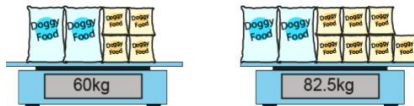
	4	2	6	6
2	8	5	13	12

Th	H	T	O
1000 1000	100 100	10 10	1 1
1000 1000	100 100	10 10	1 1
1000 1000	100 100	10 10	1 1
1000 1000	100 100	10 10	1 1

Pictorial

quotient
divisor) dividend

Use their understanding of how 2 numbers can be related additively or multiplicatively to find two unknowns



$$598 \div 13 = 46$$

		4	6
13	5	9	8
-	5	2	0
		7	8
-		7	8
			0

Write key facts out

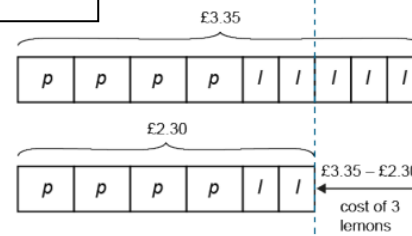
$$\begin{aligned} 13 \times 1 &= 13 & 13 \times 10 &= 130 \\ 13 \times 2 &= 26 & 13 \times 20 &= 260 \\ 13 \times 3 &= 39 & 13 \times 30 &= 390 \\ 13 \times 4 &= 52 & 13 \times 40 &= 520 \\ 13 \times 5 &= 65 & 13 \times 50 &= 650 \\ 13 \times 6 &= 78 & 13 \times 60 &= 780 \\ 13 \times 7 &= 91 & 13 \times 70 &= 910 \end{aligned}$$

($\times 40$)

($\times 6$)

Use bar modelling to solve more complex problems with 2 unknowns and 1 solution

40 children in total				
B	B	B	G	G
24 boys			16 girls	



Abstract -

quotient
divisor) dividend

Extend division box to give precise answers (decimal remainders).

$$\begin{array}{r} 27.25 \\ 4 \overline{) 1029.020} \end{array}$$

Short bus stop method

	0	4	8	9
15	7	7	13	13
		3	3	5

Choose an efficient method

Fraction remainders (lowest terms where possible)

		2	4	$\frac{4}{5}$
1	5	3	7	2
-		3	0	0
			7	2
-			6	0
			1	2

Long division using chunking

	0	4	8	9
15	7	3	3	5
-	6	0	0	0
	1	3	3	5
-	1	2	0	0
		1	3	5
-		1	3	5
				0

Mathematical Talk

Same as years before.

Chunking - I can't get any lots of 13 hundreds from 5 hundreds so we regroup and make 59 tens which is equal to 590. Now 13 multiplied by 4 equals 52 so 13 multiplied by 40 equals 520. We will subtract this from the dividend. We now have 78 ones left. 13 multiplied by 6 is equal to 78. When we subtract this from the remaining dividend, we have zero left. We took away 40 lots of 13 and then 6 lots of 13 which makes 46 lots of 13 altogether. 598 divided by 13 equals 46.

Ready to progress expectation

Understand that 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number).

Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding.

Solve problems involving ratio relationships.

Solve problems with 2 unknowns

